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Topological mass term in effective braneworld scenario with torsion

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Abstract: In this work we investigate a braneworld scenario where a Kalb-Ramond field propagates, together with gravity, in a five dimensional AdS slice. The rank-2 Kalb-Ramond field is associated with torsion. We study the compactification of the Kalb-Ramond field to four dimensional space-time without gauge fixing, focusing on the effects of torsion. On the brane the Kalb-Ramond field interacts with A_μ-gauge and scalar matter fields. We analyze the propagators of this theory and as an application investigate the consistency of such a model in the presence of a cosmic string configuration. One interesting feature of this model is the presence of topological charge coming from a topological mass term that couples the gauge field A_μ with Kalb-Ramond modes.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Spontaneous Symmetry Breaking.

Contents

1. Introduction

The hierarchy problem is one of the most instigating challenges for theoretical physicists. The disparity between the electroweak and gravitational energy scales represents one of the obstacles in the search for a unified description of the fundamental interactions. On the other hand, consistency of string theory, our main present candidate to describe all fundamental interactions, requires our world to have more than four dimensions. Originally this extra dimensions were supposed to be very small (order of Plank length). However, it has been proposed recently that the solution to the hierarchy problem may come from considering some of these extra dimensions to be not so small. In the approach of [1] our space has one or more flat extra dimensions while, according to the so called Randall-Sundrum (RS) model[2], hierarchy would be explained by one large warped extra dimension. In this RS approach our 4D world is a D3-brane embedded in a 5 dimensional Anti-de Sitter (AdS) bulk. Standard model fields are confined to the 3-brane while gravity propagates in the bulk. In this scenario the hierarchy is generated by an exponential function of the compactification radius, called warp factor.

The Randall-Sundrum model is defined in a 5-dimensional AdS slice characterized by a background metric that may be written as

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2 \,, \tag{1.1}$$

where x^μ are coordinates of the four-dimensional lorentzian surfaces at constant ϕ and $\sigma(\phi) = kr_c|\phi|$. The compact coordinate ranges from $-\pi \leq \phi \leq +\pi$ with the points (x,ϕ) and $(x,-\phi)$ identified. In this approach gravity propagates in the bulk while the standard model fields propagate on the D-3 brane defined at $\phi = \pi$. The energy scales are related in such a way that TeV mass scales are produced on the brane from Plank masses through the warp factor $e^{-2kr_c\pi}$

$$m \;=\; m_0\, e^{-2kr_c\pi}\,. \tag{1.2}$$

In our case we will deal with massless fields in the bulk but this warp factor will appear in the effective couplings in four dimensions.

It has been recently proposed that torsion can play a non trivial role in the RS scenario [3]. The motivation to this approach is the fact that both gravity and torsion are aspects of geometry. Since gravity propagates in the bulk the same should be expected for torsion. In this model the authors considered source of torsion to be a Kalb-Ramond (KR) field and showed that the zero mode would be exponentially suppressed by the AdS metric warp factor assuming a very weak value on the brane. This would cause the illusion of a torsion free Universe since the massive KR modes that would have a larger coupling are very heavy. However these massive modes could show up at TeV scales.

Torsion corresponds to the anti-symmetric part of the Affine connection and shows up when one generalizes Einsteins theory of gravity in such a way that the geometry is not only characterized by the curvature [4]. Torsion can be related to coupling to fermionic fields [5], alternatively it can be associated with a scalar field gradient for bosons in scalar tensor theories [6, 7] or to a rank 2 antisymmetric Kalb-Ramond tensor field as in [3, 8].

The anti-symmetric KR tensor field was first introduced within a string theory context [9] where it is associated with massless modes. The presence of such a background in string theory has the very important implication of non commutativity of space-time [10]. The KR tensor also appear in supersymmetric field theories. More generally, p-form gauge fields appear in many supergravity models [11].

In this work we will consider a model where torsion is represented by a KR field in a Randall-Sundrum like scenario but with gauge and matter fields on a boundary D3-brane. One of the new features of our approach is that we do not fix the KR gauge previous to compactification. This makes it possible to calculate the propagators for all the KR field components. Also we will include on the brane an alternative topological mass term coupling KR and abelian vector gauge fields that preserves Lorentz invariance. Furthermore we will analyze a possible topological defect solution. In particular we will consider a cosmic string configuration [12]–[15] although not taking gravitational effects into account as we will just focus on the effect of torsion. One finds objects analogous to these topological defects in condensed matter systems (where there is no gravity) for example in flux tubes in type-II superconductors [16], or vortex filaments in a superfluid [17]. Interesting discussions of cosmic strings in brane world scenario can be found in refs [18, 19, 20] It is important to remark that KR fields have already been introduced in the cosmic string context with dilaton gravity [21, 22] and in global vortex with extra dimensions [23]. They are an important ingredient to described vortex configuration in condensed matter systems [24, 25, 16, 17]. Important features appear when the model is supersymmetric [26]–[28].

Recently the interest in studying cosmic strings has increased again mainly because of string theory. It was previously believed that fundamental superstrings were not related to the macroscopic cosmic strings. However the compactification scenarios like [2] changed this picture, as explained in [29, 30]. We will consider on the brane a gauge cosmic string. However our field configurations can be associated with a possible effective low energy string background. The idea is to understand the strucure of a cosmic string in a background involving the KR compactified field that emerges naturally from superstring treory. We include a topological mass term on the brane as a possible interaction term.

In section 2 we present the compactification scheme for the KR field in an AdS slice. In section 3 we study a model with gauge and matter fields on the brane with two kinds of coupling with the KR fields and calculate the propagators. Finally in section 4 we study a cosmic string configuration in this framework.

2. The Kalb-Ramond field in brane world space time

Let us consider a KR field $B_{\hat{\mu}\hat{\nu}}$ in the 5-dimensional bulk (1.1) with free dynamics described by

$$S = \int dx^5 \sqrt{-g_5}\frac{1}{6}\left\{H_{\hat{\mu}\hat{\nu}\hat{\rho}}H^{\hat{\mu}\hat{\nu}\hat{\rho}}\right\}, \tag{2.1}$$

where we are representing by indices with hat $\hat{\mu}, \hat{\nu}, \ldots = 0, \ldots, 4$ the bulk coordinates $(x^\mu, x^4 \equiv \phi)$ with $\mu = 0, \ldots, 3$ and g_5 is the determinant of metric (1.1) and the rank-3 antisymmetric field strength, identified with torsion is

$$H_{[\hat{\mu}\hat{\nu}\hat{\rho}]} = \partial_{\hat{\mu}}B_{\hat{\nu}\hat{\rho}} + \partial_{\hat{\nu}}B_{\hat{\rho}\hat{\mu}} + \partial_{\hat{\rho}}B_{\hat{\mu}\hat{\nu}} \equiv \partial_{[\hat{\mu}}B_{\hat{\nu}\hat{\rho}]}\,. \tag{2.2}$$

The action (2.1) is invariant under the gauge transformation

$$\delta B_{\hat{\mu}\hat{\nu}}(x,\phi) = \partial_{\hat{\mu}}\xi_{\hat{\nu}}(x,\phi) - \partial_{\hat{\nu}}\xi_{\hat{\mu}}(x,\phi)\,. \tag{2.3}$$

Now, we follow an approach similar to [3] (see also [31]) in order to find an effective action for the KR field on the four dimensional brane. The lagrangian can be decomposed as

$$H_{\hat{\mu}\hat{\nu}\hat{\rho}}H^{\hat{\mu}\hat{\nu}\hat{\rho}} = H_{\mu\nu\rho}H^{\mu\nu\rho} + 3H_{4\nu\rho}H^{4\nu\rho}\,. \tag{2.4}$$

In our approach we will rewrite $B_{4\mu}$ as

$$B_{4\mu}(x,\phi) = \partial_4 C_\mu(x,\phi) - \partial_\mu C_4(x,\phi) \tag{2.5}$$

and the gauge invariant action gets

$$\begin{aligned}\mathcal{S}_H = \int d^4x \int d\phi\, r_c e^{2\sigma}\Big\{&\eta^{\mu\alpha}\eta^{\nu\beta}\eta^{\lambda\gamma}\frac{1}{6}H_{\mu\nu\lambda}H_{\alpha\beta\gamma} - \\ &- \frac{1}{2r_c^2}e^{-2\sigma}\eta^{\mu\alpha}\eta^{\nu\beta}\left[B_{\mu\nu}\partial_\phi^2 B_{\alpha\beta} - 2C_{\mu\nu}\partial_\phi^2 B_{\alpha\beta} + C_{\mu\nu}\partial_\phi^2 C_{\alpha\beta}\right]\Big\},\end{aligned} \tag{2.6}$$

where, $C^{\mu\nu} \equiv \partial^\mu C^\nu - \partial^\nu C^\mu$. This action is invariant under

$$\begin{aligned}\delta C_\mu(x,\phi) &= \partial_\mu\omega(x,\phi) + \xi_\mu(x,\phi)\\ \delta B_{\mu\nu}(x,\phi) &= \partial_\mu\xi_\nu(x,\phi) - \partial_\nu\xi_\mu(x,\phi)\,.\end{aligned} \tag{2.7}$$

We now use a Kaluza-Klein decomposition for the 4-D components of the KR field

$$B_{\mu\nu}(x,\phi) = \sum_{n=0}^{n=\infty} B^n_{\mu\nu}(x)\frac{\chi^n(\phi)}{\sqrt{r_c}} \tag{2.8}$$

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and for C_μ and the gauge parameters

$$C_\mu(x,\phi) = \sum_{n=0}^{\infty} C_\mu^n(x)\frac{\chi^n(\phi)}{\sqrt{r_c}} \tag{2.9}$$

$$\xi_\mu(x,\phi) = \sum_{n=0}^{\infty} \xi_\mu^n(x)\frac{\chi^n(\phi)}{\sqrt{r_c}} \tag{2.10}$$

$$\omega(x,\phi) = \sum_{n=0}^{\infty} \omega^n(x)\frac{\chi^n(\phi)}{\sqrt{r_c}}\,. \tag{2.11}$$

the corresponding gauge transformations for these modes are

$$\begin{aligned} \delta C_\mu^n(x) &= \partial_\mu\omega^n(x) + \xi_\mu^n(x) \\ \delta B_{\mu\nu}^n &= \partial_\mu\xi_\nu^n(x) - \partial_\nu\xi_\mu^n(x)\,. \end{aligned} \tag{2.12}$$

Note that C_μ^n are not just abelian gauge fields. They correspond to what are normally called Stuckelberg fields.

We now follow the approach of [3, 8] of introducing normal modes satisfying

$$-\frac{1}{r_c^2}\frac{d^2\chi_n}{d\phi^2} = m_n^2\chi_n e^{2\sigma} \tag{2.13}$$

and the orthonormality condition

$$\int e^{2\sigma}\chi^m(\phi)\chi^n(\phi)d\phi = \delta^{mn}\,. \tag{2.14}$$

Then, integrating the coordinate ϕ we get the effective action

$$\begin{aligned} \mathcal{S}_H = \int d^4x \sum_{n=0}^{\infty}\Big\{ &\eta^{\mu\alpha}\eta^{\nu\beta}\eta^{\lambda\gamma}\frac{1}{6}H_{\mu\nu\lambda}^n H_{\alpha\beta\gamma}^n + \\ &+\frac{1}{2}m_n^2\eta^{\mu\alpha}\eta^{\nu\beta}\left[B_{\mu\nu}^n B_{\alpha\beta}^n - 2C_{\mu\nu}^n B_{\alpha\beta}^n - C_{\mu\nu}^n C_{\alpha\beta}^n\right]\Big\}\,, \end{aligned} \tag{2.15}$$

where $H_{\mu\nu\lambda}^n = \partial_{[\mu}B_{\nu\lambda]}^n$.

Although the modes χ^n do not show up in the effective action, in the next section we will consider the coupling of the KR fields to the brane fields. So it is important to find the explicit solutions for these modes to get their boundary values. We can re-write equation (2.13) in terms of the variables: $z_n = \frac{m_n}{k}e^{\sigma(\phi)}$ as

$$\left(z_n^2\frac{d^2}{dz_n^2} + z_n\frac{d}{dz_n} + z_n^2\right)\chi^n = 0\,. \tag{2.16}$$

The solutions, considering $e^{-kr_c\pi} \ll 1$ are

$$\chi^0 = \frac{M^{3/2}}{M_p e^{kr_c\pi}}\,, \tag{2.17}$$

for the zero mode and

$$\chi^n(z_n) = \frac{2M^{3/2} J_0(z_n) e^{kr_c\pi}}{M_P\, \pi x_n}\,, \tag{2.18}$$

for $n \geq 1$, where $x_n = z_n(\pi)$ and $M_P^2 = M^3[1 - e^{-2kr_c\pi}]/k$ relates the four dimensional Planck scale $M_P = 2 \times 10^{18}$ GeV to the (fundamental) Planck scale M [2] so that M_P is of the same order of M. The values of these modes on the boundary will be important in the next section when the KR fields will couple to abelian and matter fields there. We will represent them in the compact form

$$\chi^n(z_n(\pi)) \;=\; \chi^n(x_n) \;\equiv \chi^n_\pi\,.$$

We can see from the solution (2.17) that the (constant) zero-mode χ^0 exhibits a suppression by a large exponential factor. This was interpreted in [3] as a possible explanation for the absence of an observable torsion in our space-time. Another very interesting consequence, also discussed in [3], would be the possibility of observing effects of the massive Kaluza Klein modes through new resonances in TeV-scale accelerators.

3. Effective four dimensional theory and propagator analysis

Now we will consider the coupling of the KR field with the gauge and scalar matter fields that live on the four dimensional brane. We consider two kinds of couplings between the KR and the electromagnetic fields located on the brane at $\phi = \pi$. One comes from a topological mass term. The other comes from the covariant derivative of the scalar field that involves both the dual KR and the electromagnetic fields. Our four dimensional action is

$$\mathcal{S}_{4d} = \int dx^4 \left[-\frac{1}{2} D_\mu \Phi D^\mu \Phi^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\xi}{3}\, \epsilon^{\mu\nu\alpha\beta} A_\mu \sum_{n=0}^{\infty} \chi^n_\pi\, H^n_{\nu\alpha\beta}(x) - \frac{\lambda}{4} \left(|\Phi|^2 - \eta^2 \right)^2 \right], \tag{3.1}$$

where we just consider flat space as we are only interested in the effects of torsion. The coupling ξ has dimension of $(mass)^{1/2}$. The covariant derivative and the field strengths are given by

$$D_\mu \;=\; \partial_\mu + iqA_\mu + ig \sum_{n=0}^{\infty} \chi^n_\pi\, \tilde{H}^n_\mu \tag{3.2}$$

$$F_{\mu\nu} \;=\; \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3.3}$$

$$H^n_{\nu\alpha\beta} \;\equiv\; \partial_{[\nu} B^n_{\alpha\beta]}\,, \tag{3.4}$$

where g is a coupling constant of dimension $(mass)^{-3/2}$ and $\tilde{H}^n_\mu$ is the mode expansion of the usual 4-dimensional KR dual of the field-strength given by

$$\tilde{H}^n_\mu \;\equiv\; \frac{1}{6} \epsilon_{\mu\nu\alpha\beta} H^{n\ \nu\alpha\beta}\,. \tag{3.5}$$

The gauge transformations that leave action (3.1) invariant are

$$\begin{aligned}\Phi(x) &\to \Phi(x)e^{i\Lambda(x)}\,,\\ A_\mu(x) &\to A_\mu(x) - \partial_\mu\Lambda(x)\,,\\ B^n_{\mu\nu}(x) &\to B^n_{\mu\nu}(x) + \partial_\mu\xi^n_\nu(x) - \partial_\nu\xi^n_\mu(x)\,,\end{aligned} \tag{3.6}$$

The effective action on the brane will include also the compactified KR action (2.15).

$$S_{eff.} = S_{4d} + S_H\,. \tag{3.7}$$

It should be noted that in this article we are not considering the curvature effects because we are only interested in the effects of the presence of torsion.

Now let us compute the propagators for the gauge-field excitations, in a particular scalar field configuration. We take the scalar field Φ to acquire the non-vanishing vacuum expectation value $<|\Phi|>= \eta$ corresponding to the minimum of the potential. We parametrize Φ as

$$\Phi = [\,\Phi(x)' + \eta\,]\, e^{i\Sigma(x)}\,, \tag{3.8}$$

where Φ' is the quantum fluctuation around the ground state η. In order to compute the propagators we have to fix the gauge so as to make the action non-singular. This is accomplished by the adding gauge-fixing terms

$$\mathcal{L}_{A_\mu} = \frac{1}{2\alpha}(\partial_\mu A^\mu + 2\alpha q\eta^2\Sigma)^2\,; \tag{3.9}$$

$$\mathcal{L}_{B^n_{\mu\nu},C^n_\mu} = -\frac{1}{2\beta_n}(\partial_\mu B^{\mu\nu}_n - 2m_n^2\beta_n C^\nu_n)^2\,. \tag{3.10}$$

Note that we are choosing this terms in such a way that the crossed terms involving A_μ and Σ and also those involving $B^{\mu\nu}_n$ and C_μ are removed. So the total action in the non perturbed form of configuration (3.8) becomes

$$\begin{aligned}\mathcal{L}_K = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{6}\left(\delta^{nn'} + g^2\eta^2\chi^n_\pi\chi^{n'}_\pi\right)H^n_{\mu\nu\kappa}H^{n'\mu\nu\kappa} - \frac{1}{3}(\xi + q\,g\eta^2)\chi^n_\pi\epsilon^{\mu\nu\alpha\beta}A_\mu H^n_{\nu\alpha\beta} -\\ &-\frac{1}{2}m_n^2(B^n_{\mu\nu}B^{n\mu\nu} + C^n_{\mu\nu}C^{n\mu\nu}) + \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + 2\alpha(q\eta^2\Sigma)^2 -\\ &-q^2\eta^2 A_\mu A^\mu - \frac{1}{2\beta_n}(\partial_\mu B^{\mu\nu}_n)^2 - 2\beta_n(m_n^2 C^\nu_n)^2 - \eta^2\partial_\mu\Sigma\partial^\mu\Sigma\,.\end{aligned} \tag{3.11}$$

Note that Σ and C_μ decouple from the other fields as a consequence of the gauge fixing terms (3.9) and (3.10) used.

The action has the general form

$$\mathcal{L} = \frac{1}{2}\sum_{\alpha\beta}\mathcal{A}^\alpha\mathcal{O}_{\alpha\beta}\mathcal{A}^\beta\,, \tag{3.12}$$

where $\mathcal{A}_\alpha = (\Sigma, A_\mu, B^n_{\mu\nu}, C^n_\mu)$ and $\mathcal{O}_{\alpha\beta}$ is the wave operator. Representing the action in this form we see that calculating the propagators is equivalent to inverting the operator $\mathcal{O}$. This happens because we are only considering bilinear terms in the fields.

	$\Theta^{\alpha\nu}$	$\Omega^{\alpha\nu}$	$S^{\alpha\nu\lambda}$	$(P_b)^{\alpha\nu,\lambda\beta}$	$(P_e)^{\alpha\nu,\lambda\beta}$
$\Theta_{\mu\alpha}$	${\Theta_\mu}^\nu$	0	${S_\mu}^{\nu\lambda}$	0	0
$\Omega_{\mu\alpha}$	0	${\Omega_\mu}^\nu$	0	0	0
$S_{\lambda\mu\alpha}$	${S_{\lambda\mu}}^\nu$	0	$\Box{\Theta_\mu}^\nu$	$2{S_\mu}^{\nu\beta}$	0
$(P_b)_{\beta\lambda,\alpha\mu}$	0	0	$2{S_\beta}^\nu{}_\mu$	${\Theta_\mu}^\nu$	0
$(P_e)_{\beta\lambda,\alpha\mu}$	0	0	0	0	${\Omega_\mu}^\nu$

Table 1: Multiplicative table.

In order to invert the operator $\mathcal{O}$ we need to use an extension of the spin-projection operator formalism presented in [32, 33]. In the present case we have to add other new operators coming from the KR terms. The two projector operators which act on the tensor field are:

$$(P_b^1)_{\mu\nu,\rho\sigma} = \frac{1}{2}(\Theta_{\mu\rho}\Theta_{\nu\sigma} - \Theta_{\mu\sigma}\Theta_{\nu\rho}) \,, \tag{3.13}$$

$$(P_e^1)_{\mu\nu,\rho\sigma} = \frac{1}{2}(\Theta_{\mu\rho}\Omega_{\nu\sigma} - \Theta_{\mu\sigma}\Omega_{\nu\rho} - \Theta_{\nu\rho}\Omega_{\mu\sigma} + \Theta_{\nu\sigma}\Omega_{\mu\rho}) \,, \tag{3.14}$$

where $\Theta_{\mu\nu}$ and $\Omega_{\mu\nu}$ are, respectively, the transverse and longitudinal projection operators, given by:

$$\Theta_{\mu\nu} = \eta_{\mu\nu} - \Omega_{\mu\nu} \,, \tag{3.15}$$

and

$$\Omega_{\mu\nu} = \frac{\partial_\mu\partial_\nu}{\Box} \,. \tag{3.16}$$

We will need also the antisymmetric operator

$$S_{\mu\gamma k} = \epsilon_{\lambda\mu\gamma k}\partial^\lambda \,. \tag{3.17}$$

By using these operators the lagrangian can be written as

$$\begin{aligned}\mathcal{L} = {} & A^\mu\left[\frac{1}{2}(\Box+2q^2\eta^2)\Theta_{\mu\nu} - \frac{1}{2}\left(\frac{1}{\alpha}\Box-2q^2\eta^2\right)\Omega_{\mu\nu}\right]A^\nu + A^\mu\left[(\xi + q\,g\eta^2)\chi_\pi^n S_{\mu\nu k}\right]B^{\nu k\ n} + \\ & + B^{\alpha\beta n}\left[(m_n^2\delta^{nn'} + (\delta^{nn'} + g^2\eta^2\chi_\pi^n\chi_\pi^{n'})\Box)\left(P_b^1\right)_{\alpha\beta.\nu k} + \left(m_n^2 + \frac{1}{4\beta_n}\Box\right)\left(P_e^1\right)_{\alpha\beta.\nu k}\right]B^{\nu k\ n'} + \\ & + \Sigma\left[2\alpha\eta^4q^2 - \eta^2\Box\right]\Sigma - m_n^2C^\mu\left[(\Box + 2m_n^4\beta_n)\Theta_{\mu\nu} + 2m_n^4\beta_n\Omega_{\mu\nu}\right]C^\nu \,.\end{aligned} \tag{3.18}$$

In order to find the wave operator′s inverse, we will need also the products of operators for all non-trivial combinations involving the projectors. The relevant multiplication rules are shown in table 1. The part of the wave operator $\mathcal{O}$ eq. (3.12) that corresponds to the gauge fields $A_\mu, B^n_{\mu\nu}$ can be split into four sectors, according to:

$$\mathcal{O}' = \begin{pmatrix} \mathcal{O}_{AA} & \mathcal{O}_{AB} \\ \mathcal{O}_{BA} & \mathcal{O}_{BB} \end{pmatrix} , \tag{3.19}$$

with

$$
\begin{aligned}
\mathcal{O}_{A_\mu A_\nu} &= a_1\Theta_{\mu\nu} + a_2\Omega_{\mu\nu} \\
\mathcal{O}_{A_\mu B^n_{\nu\kappa}} &= a_3^n S_{\mu\nu\kappa} \\
\mathcal{O}_{B^n_{\mu\nu} A_\kappa} &= -a_3^n S_{\mu\nu\kappa} \\
\mathcal{O}_{B^n_{\alpha\beta} B^{n'}_{\nu\kappa}} &= a_4^{nn'}(P_b^1)_{\alpha\beta,\nu\kappa} + a_5^{nn'}(P_e^1)_{\alpha\beta,\nu\kappa}\,,
\end{aligned}
\tag{3.20}
$$

where $a_1, \ldots, a_5$ are:

$$
\begin{aligned}
a_1 &= \frac{1}{2}(\Box + 2q^2\eta^2)\,, \\
a_2 &= -\frac{1}{2}(\frac{1}{\alpha}\Box - 2q^2\eta^2)\,, \\
a_3^n &= \frac{1}{2}(\xi + q\,g\eta^2)\chi_\pi^n\,, \\
a_4^{nn'} &= m_n^2\delta^{nn'} + (\delta^{nn'} + g^2\eta^2\chi_\pi^n\chi_\pi^{n'})\Box\,, \\
a_5^{nn'} &= (m_n^2 + \frac{1}{4\beta_n}\Box)\delta^{nn'}\,,
\end{aligned}
\tag{3.21}
$$

After some algebraic calculation, we find the inverse of the operator $\mathcal{O}'$ of eq. (3.19).

$$
\mathcal{O}'^{\,-1} = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix},
\tag{3.22}
$$

where the quantities X, Y, Z and W are:

$$
\begin{aligned}
X &= (\mathcal{O}_{AA} - \mathcal{O}_{AB}\mathcal{O}_{BB}^{-1}\mathcal{O}_{BA})^{-1}\,, \\
Z &= -\mathcal{O}_{BB}^{-1}\mathcal{O}_{BA}X\,, \\
W &= (\mathcal{O}_{BB} - \mathcal{O}_{BA}\mathcal{O}_{AA}^{-1}\mathcal{O}_{AB})^{-1}\,, \\
Y &= -\mathcal{O}_{AA}^{-1}\mathcal{O}_{AB}W\,,
\end{aligned}
\tag{3.23}
$$

or, explicitly

$$
\langle A_\mu A_\nu\rangle = \frac{i}{a_1 - 2a_3^n(a_4^{nn'})^{-1}a_3^{n'}\Box}\Theta_{\mu\nu} + \frac{i}{a_2}\Omega_{\mu\nu}
\tag{3.24}
$$

$$
\langle A_\mu B^n_{\nu\kappa}\rangle = -2a_3^n(a_4^{n'n''})^{-1}\frac{i}{a_4^{n''n'} - 2a_3^{n''}a_1^{-1}a_3^{n'}\Box}\epsilon^\lambda{}_{\mu\nu}{}^\alpha\,\partial_\lambda\Theta_{\alpha\kappa}
\tag{3.25}
$$

$$
\langle B^n_{\alpha\beta}B^{n'}_{\gamma\kappa}\rangle = \frac{i}{a_4^{nn'} - 2a_3^n a_1^{-1}a_3^{n'}\Box}(P_b^1)_{\alpha\beta\gamma\kappa} + \frac{i}{a_5^{nn'}}(P_e^1)_{\alpha\beta\gamma\kappa}\,.
\tag{3.26}
$$

As the other sectors of the lagrangian are diagonal we can calculate the other propagators by simply inverting the corresponding matrix elements. For the $\Sigma - \Sigma$ propagator we find

$$
\langle\Sigma\Sigma\rangle = \left[2\alpha\eta^3 a_2\right]^{-1}\,.
\tag{3.27}
$$

For the C_μ^n fields the the operator $\mathcal{O}$ can be written as

$$
\mathcal{O}_{C^n_\mu C^{n'}_\nu} = a_6^{nn'}\Theta_{\mu\nu} + a_7^{nn'}\Omega_{\mu\nu}\,,
\tag{3.28}
$$

where

$$\begin{aligned} a_6^{nn'} &= -m_n^2(\Box + 2m_n^4\beta_n)\delta^{nn'} \\ a_7^{nn'} &= -2m_n^6\beta_n\,\delta^{nn'} \end{aligned} \tag{3.29}$$

in this expressions there is no sum over the index n.

Inverting the operator (3.28) we find the propagator for the C_μ fields

$$< C_\mu^n C_\nu^{n'} >= \frac{i}{a_6^{nn'}}\Theta_{\mu\nu} + \frac{i}{a_7^{nn'}}\Omega_{\mu\nu}\,. \tag{3.30}$$

Let us now look for the zeros of the propagators that correspond to the effective masses. For the A_μ field the poles of eq. (3.24) come from the zeros of the operator

$$a_1 - 2a_3^n(a_4^{nn'})^{-1}a_3^{n'}\Box \tag{3.31}$$

while for the $B_{\mu\nu}^n$ fields the poles of eq. (3.26)come from the zeros of

$$a_4^{nn'} - 2a_3^n a_1^{-1}a_3^{n'}\Box\,. \tag{3.32}$$

Considering only the mode $n = 0$ of the KR field we find from equation (3.31) the momentum space poles of the propagator of the gauge field A_μ.

$$K^2 = \left[2q^2\eta^2 - \frac{(\xi + q\,g\,\eta^2\,)^2\chi_\pi^0\chi_\pi^0}{(1 + g^2\eta^2\,\chi_\pi^0\chi_\pi^0)}\right]. \tag{3.33}$$

Similarly, for the propagator KR field we find from eq. (3.32)

$$K^2 = 0 \tag{3.34}$$

$$K^2 = \left[2q^2\eta^2 - \frac{(\xi + q\,g\,\eta^2\,)^2\chi_\pi^0\chi_\pi^0}{(1 + g^2\eta^2\,\chi_\pi^0\chi_\pi^0)}\right], \tag{3.35}$$

where again $\chi_\pi^0 = \frac{M^{3/2}}{M_P}e^{-kr_c\pi}$. So we see that for the KR field there is a massless solution (3.34) that may give rise to long range effects and a massive solution (3.35) with the same mass of the A_μ gauge field.

4. Cosmic string configuration with Maxwell-Kalb-Ramond term in brane world effective theory

In this section we study a cosmic string in the present Randall-Sundrum scenario. The model we will consider is a static cosmic string configuration living on the boundary D-3 brane in the presence of the AdS bulk KR torsion background. The effective action is

$$\begin{aligned} \mathcal{S}_M = \int dx^4\Bigg[&-\frac{1}{2}D_\mu\Phi D^\mu\Phi^* - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{6}\sum_{n=0}^{\infty} H_{\mu\nu\lambda}^n H^{\mu\nu\lambda\,n} - \frac{\xi}{3}\epsilon^{\mu\nu\alpha\beta}A_\mu\sum_{n=0}^{\infty}\chi_\pi^n H_{\nu\alpha\beta}^n - \\ &-\frac{1}{2}\sum_{n=0}^{\infty} m_n^2\eta^{\mu\alpha}\eta^{\nu\beta}\left[B_{\mu\nu}^n B_{\alpha\beta}^n - 2C_{\mu\nu}^n B_{\alpha\beta}^n + C_{\mu\nu}^n C_{\alpha\beta}^n\right] - \frac{\lambda}{4}(|\Phi|^2 - \eta^2)^2\Bigg]. \end{aligned} \tag{4.1}$$

The cosmic string configuration for the scalar field [34] can be parametrized as usual in cylindrical coordinates (t, r, θ, z), where $r \geq 0$ and $0 \leq \theta < 2\pi$

$$\Phi = \varphi(r) e^{i\theta}\,. \tag{4.2}$$

The boundary conditions for the fields φ are

$$\begin{aligned} \varphi(r) &= \eta \qquad r \to \infty \\ \varphi(r) &= 0 \qquad r = 0 \end{aligned} \tag{4.3}$$

The configuration for the gauge fields will be fixed by the condition that the energy must be finite. This implies that at spatial infinity ($r \to \infty$) we must have

$$D_\theta \Phi = \frac{1}{r}\left(\frac{\partial \Phi}{\partial \theta}\right) + \frac{1}{r}\left(iqA_\theta + ig\sum_{n=0}^{\infty} \chi_\pi^n\, \tilde{H}_\theta^n\right)\Phi = 0 \tag{4.4}$$

we take the following ansatz for the $\theta-$ component of the gauge fields

$$\begin{aligned} A_\theta &= \frac{1}{q}(P(r) - 1) \\ \sum_{n=0}^{\infty} \chi_\pi^n\, \tilde{H}_\theta^n &= \frac{H(r)}{gr}\,. \end{aligned} \tag{4.5}$$

The condition (4.4) will be satisfied if

$$\begin{aligned} P(r) &= 0 \qquad (r \to \infty) \\ P(r) &= 1 \qquad (r = 0) \end{aligned} \tag{4.6}$$

$$\begin{aligned} H(r) &= 0 \qquad (r \to \infty) \\ H(r) &= 0 \qquad (r = 0)\,. \end{aligned} \tag{4.7}$$

Without the KR field this configuration would be the ordinary one for a non charged static cosmic string that does not allow electric charge [34]. Now let us analyze how the presence of the KR field changes this picture. For this we need the temporal component of the A_μ and $\tilde{H}_\mu$ fields. We take the ansatz that they depend only on r: $A_t = A_t(r)$, $\sum_{n=0}^{\infty} \chi_\pi^n\, \tilde{H}_t^n = \left(\sum_{n=0}^{\infty} \chi_\pi^n\, \tilde{H}_t^n\right)(r)$. The finite energy condition for this component in the static case reads

$$D_t \Phi = \left(iqA_t + ig\sum_{n=0}^{\infty} \chi_\pi^n\, \tilde{H}_t^n\right)\Phi = 0 \tag{4.8}$$

these equation is satisfied if the fields have the following asymptotic behavior

$$\begin{aligned} A_t &= 0 \qquad (r \to \infty) \\ A_t &= a \qquad (r = 0) \end{aligned} \tag{4.9}$$

$$\begin{aligned} \sum_{n=0}^{\infty} \chi_\pi^n\, \tilde{H}_t^n &= 0 \qquad (r \to \infty) \\ \sum_{n=0}^{\infty} \chi_\pi^n\, \tilde{H}_t^n &= h \qquad (r = 0)\,. \end{aligned} \tag{4.10}$$

We will consider in this first approach to such a model that the background configuration does not involve the fields C_μ^n. That means: our ansatz involves $C_\mu^n = 0$.

In order to simplify the form of the equations of motion, let us use the notation of the electric and magnetic fields, generalized also to the KR fields. The electrical type field E^i and magnetic type field B^i are defined as usual as

$$E^i = F^{0i} \qquad B^i = -\epsilon^{ijk} F_{jk}\,, \tag{4.11}$$

where $i, j, k = 1, 2, 3$. For the KR we define

$$\mathcal{E}^{(n)i} = -\epsilon^{ijk} H^{(n)}_{0jk} \qquad \mathcal{B}^{(n)} = \epsilon^{ijk} H^{(n)}_{ijk} \tag{4.12}$$

which give us: $\tilde{H}^{(n)\mu} = (\mathcal{B}^{(n)}, \vec{\mathcal{E}}^{(n)})$.

The equations of motion for the gauge fields, in terms of (4.11) and (4.12) get

$$\vec{\nabla} \times \vec{E}(r) = 0 \tag{4.13}$$
$$\vec{\nabla} \cdot \vec{B}(r) = 0 \tag{4.14}$$
$$\vec{\nabla} \cdot \vec{E} = -2\xi \sum_{n=0}^{\infty} \chi^n_\pi \mathcal{B}^n(r) + \rho(r) \tag{4.15}$$
$$\vec{\nabla} \times \vec{B}(r) = -2\xi \sum_{n=0}^{\infty} \chi^n_\pi \vec{\mathcal{E}}^n(r) + \vec{j}(r) \tag{4.16}$$

and the topological KR fields.

$$\vec{\nabla} \cdot \vec{\mathcal{E}}^n(r) = 0 \tag{4.17}$$
$$\vec{\nabla} \times \vec{\mathcal{E}}^n(r) = \xi\, \chi^n_\pi\, \vec{B} + 3\, m_n^2\, \vec{\mathcal{P}}^n + \frac{g\chi^n_\pi}{q}\, \nabla \times \vec{j}(r) \tag{4.18}$$
$$\vec{\nabla}\mathcal{B}^n(r) = \xi \chi^n_\pi \vec{E} + 3 m_n^2 \vec{\mathcal{H}}^n + \frac{g\chi^n_\pi}{q} \nabla\rho \tag{4.19}$$

where $\vec{\mathcal{P}}^n = B^n_{0i}$, $\vec{\mathcal{H}}^n = \epsilon_{ijk} B^n_{ij}$ and the current $j^\mu = (\rho, \vec{j})$

$$j_\mu = -\frac{iq}{2}(\Phi^* D_\mu \Phi - \Phi D_\mu \Phi^*)\,. \tag{4.20}$$

The topological current, divergenceless off shell, is given by

$$K^{\mu\,(n)} = \partial_\nu(\tilde{F}^{\mu\nu} + \tilde{B}^{\mu\nu\,(n)})\,. \tag{4.21}$$

However, due to the absence of magnetic monopoles and to the zero order the first term can be thrown away and we get

$$K^{\mu\,(n)} = \partial_\nu \tilde{B}^{\mu\nu\,(n)}\,, \tag{4.22}$$
$$\partial_\mu K^{\mu\,(n)} = 0\,. \tag{4.23}$$

The corresponding topological charged is defined as

$$Q^{(n)}_T \equiv \int K^{(n)}_t d^3x = \int d^3x\, \mathcal{B}^{(n)}\,, \tag{4.24}$$

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where $\mathcal{B}^{(n)}$ stands for the magnetic-like field (a scalar) associated to the 2-form potential. In order to study the topological charge effect associated with the KR field we can write the $\mathcal{B}^{(n)}$ field as

$$\mathcal{B}^{(n)} = \vec{\nabla} \cdot \vec{\mathcal{P}}^{(n)} . \qquad (4.25)$$

Then using definition (4.24) and the divergence theorem in eq. (4.15), we define

$$Q_T \equiv 2\,\xi \sum_{n=0}^{\infty} \chi_\pi^n Q_T^{(n)} = 2\,\xi \sum_{n=0}^{\infty} \chi_\pi^n \int_V d^3x \vec{\nabla}.\vec{\mathcal{P}}^{(n)} = 2\,\xi \sum_{n=0}^{\infty} \chi_\pi^n \int_S \vec{\mathcal{P}}^{(n)}.\hat{u}\ da, = Q\,, \quad (4.26)$$

where Q stands for the electric charge

$$Q = \int d^3x\rho = q \int d^3x \left(qA_t + g \sum_{n=0}^{\infty} \chi_\pi^n \tilde{H}_t^n \right) \varphi^2 . \qquad (4.27)$$

From eq. (4.26) we find the behavior of the field $\mathcal{P}$

$$\mathcal{P}(r) \equiv 2\,\xi \sum_{n=0}^{\infty} \chi_\pi^n \mathcal{P}_r^n = \frac{1}{2\pi}\frac{Q}{r} . \qquad (4.28)$$

However, equation (4.18) tell us that, since outside the cosmic string $\vec{\mathcal{E}}^n$, $\vec{B}$ and $\vec{j}$ vanish, all the massive modes have a vanishing $\vec{\mathcal{P}}^n$ also. So that only the massless mode has long range effects. That means: outside the cosmic string:

$$\mathcal{P}(r) \equiv 2\,\xi\, \chi_\pi^0 \mathcal{P}_r^0 = \frac{1}{2\pi}\frac{Q}{r} . \qquad (4.29)$$

This result shows that there is a field configuration outside the cosmic string that may cause some non trivial effects.

5. Conclusion

In this work, we analyzed the KR field in a five dimensional brane world scenario where it is associated with torsion. The theory is compactified to four dimensions without gauge fixing. As a result of this process we have a massless and a tower of massive modes corresponding to four dimensional Kalb-Ramond fields and also Stuckelberg fields on the brane. After the compactification we consider an effective field theory on the brane including the free Kalb-Ramond sector plus matter and U(1) gauge fields. Motivated by the interest in studying the effect of this Kalb-Ramond torsion background in a cosmic string configuration we included in the model two types of Kalb-Ramond interactions, one in the U(1)-group covariant derivative and the other given by a topological mass term.

We also analyzed the propagators of this theory and found the Kalb-Ramond mass contribution to the A_μ-gauge field caused by the topological mass term considering the zero mode. Finally we showed that it is possible to construct a topological configuration like a static cosmic string on the brane whose formation involves only the zero mode.

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